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FASST Soil Moisture, Soil Temperature: Original Versus New

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Abstract: This paper discusses only the differences between the original version of FASST (Frankenstein and Koenig 2004a, 2004b) and the new version. This report is intended as a supplement to the original model documentation. In its original incarnation, energy and mass transport associated with water vapor in the soil matrix were ignored. The author added these so that model usage could be expanded to include biological investigations yet still retain its initial focus of soil strength, and sensor performance inputs. Also ignored in the original version was water transport due to soil temperature gradients. In determining the new soil temperatures and moistures, the original model first achieved convergence in the temperature profile followed by the moisture profile at a given time step. The new version of FASST solves both of these sets of equations simultaneously. No changes have been made to the equations describing the canopy physical state except to allow for mixed precipitation.

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Nomenclature

f	subscript indicating low foliage (shrubs, grass, crops, etc., vegetation) terms
g	subscript indicating ground terms
α_f, α_g	shortwave albedo (0 – 1)
α_{vG}	van Genuchten pressure head constant (cm^{-1})
$\alpha_{t,j}$	numerical calculation constant
$\beta_{t,j}$	numerical calculation constant
$\delta_{t,j}$	numerical calculation constant
Δt	time step (*sec*)
Δz	node thickness (*m*)
ε_f , ε_g	longwave emissivity (0 – 1)
ε_1	calculation variable ($\varepsilon_1 = \varepsilon_f + \varepsilon_g - \varepsilon_f \varepsilon_g$)
$\phi_{t,j}$	numerical calculation constant (*K*)
γ	soil water surface tension (g/s^2) $\gamma = 75.6 - 0.1425(T - 273.15) - 2.38x10^{-4}(T - 273.15)^2$
γ_d	bulk dry density of soil (kg/m^3)
γ_p	precipitation density (kg/m^3)
$\gamma_{t,j}$	numerical calculation constant
γ_0	soil water surface tension at 25 °*C* (71.89 g/s^2)
η	mechanistic enhancement factor for vapor flow
Γ_e	latent heat exchange stability correction factor
Γ_h	sensible heat exchange stability correction factor

Γ_m momentum stability correction factor

κ thermal conductivity ($W/m{\cdot}K$)

κ_a thermal conductivity of dry air ($W/m{\cdot}K$)

κ_i thermal conductivity of ice ($W/m{\cdot}K$)

κ_v thermal conductivity of water vapor ($W/m{\cdot}K$)

κ_{veg} vegetation thermal conductivity (κ_{veg} = 0.38 $W/m{\cdot}K$)

κ_w thermal conductivity of water ($W/m{\cdot}K$)

υ_v vertical rate of water vapor flow (m/s)

υ_w vertical rate of water flow (m/s)

$\mu_{j,i}$ numerical calculation constant

θ_i volumetric ice content (cm^3/cm^3)

θ_{max} maximum soil moisture content (m^3/m^3)

θ_r residual soil moisture content (m^3/m^3)

θ_v volumetric vapor content (cm^3/cm^3)

θ_w volumetric water content (cm^3/cm^3)

ρ_a air density at the instrument height (kg/m^3)

ρ_{af} density of air in the atmosphere/foliage interface $\rho_{af} = 0.5\left(\rho_a + \rho_f\right)$ (kg/m^3)

ρ_{ag} density of air at the foliage/ground interface (kg/m^3)

ρ_f air density in the foliage (kg/m^3)

ρ_i density of ice (kg/m^3)

ρ_v density of water vapor (kg/m^3)

ρ_{vs} saturated water vapor density (kg/m^3)

ρ_w density of water (kg/m^3)

σ Stefan-Boltzman constant (5.699e-08 $W/m^2{\cdot}K^4$)

σ_f	foliage fractional coverage
Ψ	pressure head (m)
c_p	specific heat of the soil ($J/kg{\cdot}K$)
$c_{p,a}$	specific heat of air ($J/kg{\cdot}K$)
$c_{p,i}$	specific heat of ice ($-13.3+7.80T_a$ $J/kg{\cdot}K$)
$c_{p,p}$	precipitation specific heat ($J/kg{\cdot}K$)
$c_{p,s}$	specific heat of dry soil ($J/kg{\cdot}K$)
$c_{p,v}$	specific heat of water vapor ($J/kg{\cdot}K$)
$c_{p,veg}$	specific heat of vegetation (3500 σ_f $J/kg{\cdot}K$)
$c_{p,w}$	specific heat of water ($J/kg{\cdot}K$)
C_e	empirical coefficient associated with Γ_e
C_e^g	bulk transfer coefficient for latent heat near the ground
C_{en}^g	bulk transfer coefficient near the ground for near-neutral conditions
C_f	bulk transfer coefficient for turbulent heat in the foliage
C_h	empirical coefficient associated with Γ_h
C_h^g	bulk transfer coefficient for sensible heat near the ground
C_{hn}^f	bulk transfer coefficient for latent heat at the top of the foliage for near-neutral conditions $C_{hn}^f=\left[k/\ln\left(\frac{Z_a-Z_d}{z_0^f}\right)\right]^2$
C_{hn}^g	bulk transfer coefficient for sensible heat near the ground for near-neutral conditions
C_m	empirical coefficient associated with Γ_m
C_{ng}^0	bulk transfer coefficient for momentum near the ground for near-neutral conditions

C_r	condensation rate
D	molecular diffusivity of water vapor in air (m^2/s)
D_h	molecular thermal diffusion coefficient (m^2/s)
D_r, D_s	vegetation dripped rain, snow (m/s)
D_v	molecular diffusion coefficient of water vapor (m^2/s)
e_o	windless sensible heat correction factor (2.0 W/m^2)
E	evaporation rate (m/s)
f_1, f_2, f_3	variables used to calculate stomatal resistance
F_f	sum of energy terms at the atmosphere/foliage interface (W/m^2)
g	gravity (9.81 m^2/s)
h	total head (m) [$h = z - \Psi = z - P_a / \rho_w g$]
h_{pond}	head due to water collecting on the surface (m)
$h_{i,melt}$	head due to melting ice (m)
$h_{s,melt}$	head due to melting snow (m)
H_f	sensible heat at the atmosphere/foliage interface (W/m^2) $H_f = \left(e_0 + 1.1LAI\rho_{af}c_{p,a}C_f W_{af}\right)(T_{af} - T_f)$
H_g	sensible heat at the foliage/ground interface (W/m^2) $H_g = \left(e_0 + \rho_{ag}c_{p,a}C_h^g W_{af}\right)(T_{af} - T_g)$
H_R	relative humidity of the soil (0 – 1)
$I_{ir}^{\downarrow}$	total incoming infrared radiation (W/m^2)
I_r, I_s	vegetation intercepted rain, snow (m/s)
$I_s^{\downarrow}$	total incoming solar radiation
k	von Karmen's constant (k = 0.4)
k_{th}	soil thermal diffusivity ($k_{th} = \kappa/c_p$ m^2/s)
K_{lh}	pressure-driven hydraulic conductivity (m/s)
K_{lT}	temperature-driven hydraulic conductivity ($m^2/K \cdot s$)

K_{sat} saturated hydraulic conductivity (m/s)

K_{vh} pressure-driven vapor conductivity (m/s)

K_{vT} temperature-driven vapor conductivity ($m^2/K \cdot s$)

l latent heat (J/kg)

l_{fus} latent heat of fusion (J/kg)

l_{sub} latent heat of sublimation (2.838e06 J/kg)

l_w latent heat of vaporization (J/kg)

L_f latent heat at the atmosphere/foliage interface (W/m^2)

$$L_f = LAI \rho_{af} C_f l W_{af} r''(q_{af} - q_{f,sat})$$

L_g latent heat at the foliage/ground interface (W/m^2)

$$L_g = C_e^g l W_{af} \rho_{ag} (q_{af} - q_g)$$

LAI foliage Leaf Area Index (m^2/m^2)

$$LAI = LAI_{\min} + \left(1.0 - 0.0016\left[298.0 - T_g\right]^2\right)\left[LAI_{\max} - LAI_{\min}\right]$$

LAI_{min} minimum foliage Leaf Area Index (m^2/m^2)

LAI_{max} maximum foliage Leaf Area Index (m^2/m^2)

m_c soil clay fraction (0 – 1)

m_{vG} van Genuchten constant ($m_{vG} = 1 - 1/n_{vG}$)

M molecular weight of water (0.018015 kg/mol)

n soil porosity (0 – 1)

n_{vG} van Genuchten constant

P_a pressure (Pa)

P_f precipitation heat at the atmosphere/foliage interface (W/m^2)

$$P_f = -\gamma_p \left[1.0 - \exp\left\{-0.5\left(LAI + SAI\right)\right\}\right] P_r c_p T_p$$

P_g precipitation heat at the foliage/ground interface (W/m^2)

$$P_g = -\gamma_p \left(P_r - Interception + Drip\right) c_{p,p} T_p \ (m/s)$$

P_r precipitation rate (m/s)

q_a mixing ratio of air above the foliage

q_{af} mixing ratio of the air at the foliage interface

$$q_{af} = \frac{[(1-\sigma_f)q_a + \sigma_f(0.3q_a + 0.6q_{f,sat}r'' + 0.1q_{g,sat}H_R)]}{1-\sigma_f[0.6(1-r'') + 0.1(1-H_R)]}$$

q_f mixing ratio of air at the top of the foliage

q_g mixing ratio of air at the ground surface

$q_{f,sat}$ saturated foliage mixing ratio

$q_{g,sat}$ saturated ground mixing ratio

q_{top} surface water flux (m/s)

r_a atmospheric resistance to water vapor diffusion $r_a = 1/C_f W_{af}$ (s/m)

r_{ce} turbulent Prandtl number (r_{ce} = 0.71)

r_{ch} turbulent Schmidt number (r_{ch} = 0.63)

r_s stomatal resistance to vapor diffusion $r_s = \frac{r_{s,\min}}{LAI} f_1 f_2 f_3$ (s/m)

$r_{s,min}$ minimum stomatal resistance to vapor diffusion (s/m)

r'' foliage surface wetness factor $r'' = r_a\left(r_a + r_s\right)^{-1}$

R universal gas constant for air (8.314 $J/mol{\cdot}K$)

R_{ib} bulk Richardson number

SAI Stem area index

t time (s)

T temperature (K)

T_a air temperature (K)

T_{af} air temperature in the foliage

$$T_{af} = (1-\sigma_f)T_a + \sigma_f(0.3T_a + 0.6T_f + 0.1T_g) \ (K)$$

T_f, T_g foliage, ground surface temperature (K)

T_p precipitation temperature (K)

u_* friction velocity (m/s)

U_p	mass precipitation flux ($kg/m^2 \cdot s$)
V_v	vapor flow rate (m/s)
V_w	water flow rate (m/s)
w	relative soil moisture
W	wind speed at the instrument height Z_a (m/s)
W_{af}	wind speed at the air/foliage interface $W_{af} = 0.83\sigma_f W' \sqrt{C_{hn}^f} + (1-\sigma_f)W'$ (m/s)
W'	adjusted wind speed (W' = 2.0 m/s if W is below 2.0 m/s)
z	depth (m)
z_o^f	foliage roughness length (m)
z_o^g	ground roughness length (m)
z_o^h	sensible heat ground roughness length (m)
z_o^q	latent heat ground roughness length (m)
Z_a	shelter/instrument height for air temperature (m)
Z_d	zero displacement height $Z_d = 0.701 Z_f^{0.975}$ (m)
Z_f	vegetation height (m)
Z_{rh}	height above the ground of the relative humidity measurement (m)
Z_u	height of the measured wind speed (m)

Preface

This technical report was prepared by Susan Frankenstein, Research Physical Scientist, Terrestrial and Cryospheric Sciences Branch, US Army Engineer Research and Development Center (ERDC), Cold Regions Research and Engineering Laboratory (CRREL), Hanover, New Hampshire.

The author thanks Geoff Koenig and Connie Scott for proofreading the report and “test driving” the model. The author also thanks Greg Fall at NOAA’s NOHRSC for helping to debug the new version.

This report was prepared under the general supervision of Dr. Mark L. Moran, Chief, Terrestrial and Cryospheric Sciences Branch; Dr. Justin B. Berman, Chief, Research and Engineering Division, CRREL; and Dr. Robert E. Davis, Director, CRREL.

At the time this work was performed, Colonel Richard B. Jenkins was Commander and Executive Director of ERDC. Dr. James R. Houston was Director.

1.0 Introduction

This paper discusses only the differences between the original version of FASST (Frankenstein and Koenig 2004a, 2004b) and the new version. This report is intended as a supplement to the original model documentation. In its original incarnation, energy and mass transport associated with water vapor in the soil matrix were ignored. I added these so that model usage could be expanded to include biological investigations yet still retain its initial focus of soil strength, and sensor performance inputs. Also ignored in the original version was water transport due to soil temperature gradients. In determining the new soil temperatures and moistures, the original model first achieved convergence in the temperature profile followed by the moisture profile at a given time step. The new version of FASST solves both of these sets of equations simultaneously. No changes have been made to the equations describing the canopy physical state except to allow for mixed precipitation.

To begin, section 2.0 provides a brief synopsis of the original FASST governing equations and solution procedures that are different in the new version. Readers desiring more details should refer to Frankenstein and Koenig (2004a, 2004b). Section 3.0 contains the new version of the equations and solution procedures. A complete list of all parameters used can be found in the beginning of this report.

2.0 Original

2.1 Temperature/Energy

The temperature gradient in a non-uniform soil layer can be described by the one dimensional heat flow equation

$$\frac{\partial T}{\partial t}-\frac{l_{fus}}{c_p}\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t}=\frac{\partial}{\partial z}\left(\frac{\kappa}{c_p}\frac{\partial T}{\partial z}\right)-v_w\frac{c_{p,w}}{c_p}\frac{\partial T}{\partial z} \tag{2.1}$$

where T is the temperature (K), t is time (s), κ is the thermal conductivity ($W/m{\cdot}K$), $c_{p,w}$ is the specific heat of water ($J/kg{\cdot}K$), c_p is the specific heat of the soil ($J/kg{\cdot}K$), v_w is the vertical rate of water flow (m/s), l_{fus} is the latent heat of fusion (J/kg), θ_i is the volumetric ice content (cm^3/cm^3), ρ_i is the density of ice (kg/m^3), ρ_w is the density of water (kg/m^3), and z is depth (m) measured positive downward from the surface. The second term on the left-hand side of equation (2.1) represents heat lost/gained due to ice formation/melting and the terms on the right-hand side incorporate temperature changes due to vertical heat conduction and water flow, respectively.

If low vegetation is present, the atmosphere/foliage energy exchange is given as

$$F_f=0=\sigma_f\left[I_s^{\downarrow}(1-\alpha_f)+\varepsilon_f I_{ir}^{\downarrow}-\varepsilon_f\sigma T_f^4-P_f\right]+\frac{\sigma_f\varepsilon_f\varepsilon_g\sigma}{\varepsilon_1}(T_g^4-T_f^4)+H_f+L_f\,. \tag{2.2}$$

T_f is the foliage temperature (K), T_g is the ground temperature (K), ε_g is the ground emissivity, σ is Stefan-Boltzman constant (5.699e-08 $W/m^2{\cdot}K^4$), $I_s^{\downarrow}$ and $I_{ir}^{\downarrow}$ are the total incoming solar and infrared radiation (W/m^2), H_f, L_f, and P_f are the sensible, latent, and precipitation heat fluxes at the foliage surface, respectively (W/m^2), and $\varepsilon_1=\varepsilon_f+\varepsilon_g-\varepsilon_f\varepsilon_g$. The foliage fractional coverage σ_f, shortwave albedo α_f, and emissivity ε_f are functions of the vegetation type (high, medium, or low) and season (winter, spring, summer, and fall).

The energy flux exchange at the foliage/ground interface as

$$F(T_g) = 0 = \left(1-\sigma_f\right)\left[I_s^{\downarrow}(1-\alpha_g) + \varepsilon_g I_{ir}^{\downarrow} - \varepsilon_g \sigma T_g^4\right] - P_g - \frac{\sigma_f \varepsilon_f \varepsilon_g \sigma}{\varepsilon_1}(T_g^4 - T_f^4)$$
$$+ H_g + L_g + \kappa \frac{\partial T_g}{\partial z} + l_{fus}\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t}\Delta z - v_w c_{p,w} T_g \qquad (2.3)$$

where $(1-\sigma_f)$ represents the radiant fluxes not intercepted by the vegetation, α_g is the shortwave albedo of the ground, and H_g, L_g, and P_g are the sensible, latent, and precipitation heat fluxes at the ground surface, respectively (W/m^2), and Δz (m) is the thickness of the top node. Both α_g and ε_g are a function of the soil type and range from 0.23 to 0.40 and 0.92 to 0.97, respectively. The third term in the second row of equation (2.3) takes care of heat conduction to/from the surface by the underlying ground, depending on the temperature gradient. This is followed by the heat released/absorbed by the soil as the soil moisture melts/freezes. Finally, the last term represents heat that is advected away from/towards the surface as a result of the vertical movement of moisture.

The first term in equations (2.2) and (2.3) represent the amount of solar, or shortwave radiation, absorbed by the surface. The second term is the absorbed incoming longwave radiation while the third term is the emitted longwave radiation. The sensible and latent heat fluxes together are called the turbulent heat fluxes and have non-zero values in the presence of wind. The precipitation heat represents the energy needed to cool or heat any snow or rain that falls on the surface. The term after the precipitation heat flux represents the radiative exchange between the foliage and ground surfaces. In equations (2.2) and (2.3), heat that is transferred to the surface is considered positive.

2.2 Moisture/Mass

The flow of water (v_w) through a porous media is governed by Darcy's Law, which states that

$$v_w = K_{lh}\frac{\partial h}{\partial z} = -K_{lh}\left(-1 + \frac{\partial \Psi}{\partial z}\right) \qquad (2.4)$$

where K_{lh} (m/s) is the pressure-driven hydraulic conductivity and h (m) the total head equals the elevation head, or depth (z), minus the pressure

head (Ψ), i.e., $h = z - \Psi = z - P_a / \rho_w g$, P_a (*Pa*) is pressure, ρ_w (kg/m^3) is the density of water, and g (9.81 m^2/s) is gravity. For unsaturated soil, $\Psi < 0$.

Also governing the flow of moisture through a soil is the conservation of mass, which states that the time rate of change of the moisture content in a given volume equals the net gain/loss of fluid in the volume, i.e.,

$$\frac{\partial \theta_w}{\partial t} = -\frac{\partial v_w}{\partial z} - \frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} + sources - losses \tag{2.5}$$

where θ_w (cm^3/cm^3) is the volumetric moisture content. Equation (2.5) assumes that changes with respect to time in the soil porosity and water density are negligible compared to changes in the soil moisture and total head. The source and loss terms in this equation account for occurrences such as runoff and plant root uptake.

Equation (2.5) is subject to the following flow boundary conditions at the surface and at the bottom of the soil column

$$\begin{aligned} q_{top} &= -E + C_r + P_r + \left(h_{pond} + h_{i,melt} + h_{s,melt}\right)/\Delta t \qquad @\ z = 0 \\ q_{bot} &= K_{lh}\sin(slope) \qquad @\ z = z_{bot} \end{aligned} \tag{2.6}$$

where E (m/s) is the evaporation rate, C_r (m/s) is the condensation rate, P_r (m/s) is the rate of precipitation, h_{pond} (m) is the head due to water collecting on the surface, and $h_{i,melt}$ (m) and $h_{s,melt}$ (m) are the heads due to melting ice and snow, respectively, and Δt (*sec*) is the time step. If the ground is sloped, no water accumulates and any water that falls on the surface, but does not infiltrate, becomes runoff.

2.3 Hydraulic Flow Parameters

The pressure-driven flow parameters are unchanged between the original and new versions of FASST. Since the temperature-controlled equivalent (see section 3.3) are related, I include them here for reference purposes. The relationship between volumetric moisture content and pressure head is highly nonlinear. As in the original version of FASST, following the work of van Genuchten (1980)

$$\theta_w = \theta_r + \frac{\theta_{\max} - \theta_r}{\left(1 + \left|\alpha_{vG}\Psi\right|^{n_{vG}}\right)^{m_{vG}}} \tag{2.7}$$

where θ_r is the residual volumetric water content, θ_{max} is the maximum volumetric water content, α_{vG} (cm^{-1}) is a constant related to the reciprocal of the bubbling pressure head, n_{vG} is a constant dependent on the distribution of pores, and $m_{vG} = 1 - 1/n_{vG}$.

The pressure-driven hydraulic conductivity, K_{lh}, is modified from the original version of FASST to take into account the decrease in flow under freezing conditions and is given as (van Genuchten 1980, Niu and Yang 2006)

$$K_{lh} = K_{sat}\left(1 + 8\theta_i\right)^2 w^{0.5}\left[1 - \left(1 - w^{1/m_{vG}}\right)^{m_{vG}}\right]^2 \qquad w = \frac{\theta_w - \theta_r}{\theta_{\max} - \theta_r}. \tag{2.8}$$

where K_{sat} (m/s) is the saturated hydraulic conductivity.

2.4 Turbulent Energy Drag Coefficients

The bulk transfer coefficient for sensible heat C_h^g (Deardorff 1978) is calculated using the bulk transfer coefficient near the ground C_{hn}^g (Koenig 1994) and at the atmosphere/foliage interface C_{hn}^f (Balick et al. 1981) for near-neutral stability plus a sensible heat exchange stability correction factor Γ_h

$$C_h^g = \Gamma_h[(1-\sigma_f)C_{hn}^g + \sigma_f C_{hn}^f]$$
$$C_{hn}^g = \frac{\left[\dfrac{k}{\ln\left(Z_a/z_o^g\right)}\right]^2}{\left[r_{ch} + \dfrac{\ln(Z_a/Z_u)}{\ln(Z_a/z_o^g)}\right]}. \tag{2.9}$$

The ground roughness length z_o^g (m) is equal to 0.001 m for all soil types and 0.0006 m for snow. Since Z_a (m), the height of the measured air temperature, equals Z_u (m), the height of the measured wind speed, C_{hn}^g reduces to

$$C_{hn}^{g}=\frac{\left[\dfrac{k}{\ln\left(Z_a/z_o^g\right)}\right]^2}{r_{ch}} \tag{2.10}$$

where the turbulent Schmidt number r_{ch} is hardwired in the code as 0.63 for all soil types, as is k, von Karmen's constant (0.4). The term Γ_h in equation (2.9) accounts for non-neutral conditions and is defined as

$$\Gamma_h=\begin{cases}\dfrac{1.0}{\left(1.0-16.0R_{ib}\right)^{0.5}} & R_{ib}<0.0\\ 1.0 & R_{ib}=0.0\\ \dfrac{1.0}{1.0-5.0R_{ib}} & 0.0<R_{ib}<0.2\end{cases} \tag{2.11}$$

$$R_{ib}=\frac{2gZ_a\left(T_{af}-T_g\right)}{\left(T_{af}+T_g\right)W_{af}^2}$$

where R_{ib} is the bulk Richardson number. Similarly for the latent heat, $\Gamma_e=\Gamma_h$ and C_{en}^{g} follows the development for C_{hn}^{g} with Z_a being replaced with Z_{rh} (m), the height above the ground of the relative humidity measurement, r_{ch} with r_{ce} the turbulent Prandtl number (0.71, hardwired in the code).

2.5 Precipitation Flux

In the original version of FASST, no mixed precipitation was allowed. Thus, for a bare surface it was defined as

$$\begin{aligned}P_g&=U_p c_{p,p} T_p\\ U_p&=-\gamma_p\cdot fallrate\end{aligned} \tag{2.12}$$

with T_p the precipitation temperature defined as the air wet-bulb temperature, $c_{p,p}$ as either the specific heat of water, $c_{p,w}$ (4217.7 $J/kg{\cdot}K$) or ice, $c_{p,i}$ ($-13.3+7.80T_a$ $J/kg{\cdot}K$) depending on T_p, U_p the mass precipitation flux ($kg/m^2{\cdot}s$), and γ_p the precipitation density (kg/m^3). The *fallrate* is in (m/s).

2.6 Solution

Since the solutions to the temperature and moisture balances are discussed in detail in Frankenstein and Koenig (2004a, 2004b), I present only enough material so that comparisons can be made with the current procedures.

2.6a Temperature

Equation (2.1) is solved using a modified second-order Crank-Nicholson approach. Following the technique presented in Hornbeck (1975), equation (2.1) is rewritten as

$$\begin{aligned}\frac{T_{t+1,j}-T_{t,j}}{\Delta t}-\frac{l_{fus}}{c_p}\frac{\rho_i}{\rho_w}\frac{(\Delta\theta_i)_{t,j}}{\Delta t}=\left[\frac{k_{th_{t,j+1}}-k_{th_{t,j}}}{\Delta z}\right]\left[\frac{T_{t,j+1}-T_{t,j}}{\Delta z}\right]+\\ \frac{k_{th_{t,j}}}{2}\left[\frac{T_{t+1,j+1}-2T_{t+1,j}+T_{t+1,j-1}}{(\Delta z)^2}+\frac{T_{t,j+1}-2T_{t,j}+T_{t,j-1}}{(\Delta z)^2}\right]\\ -\frac{c_{p,w}}{c_{p_{j,i}}}v_{w:t,j}\left[\frac{T_{t,j+1}-T_{t,j}}{\Delta z}\right]\end{aligned} \tag{2.13}$$

where $k_{th} = \kappa/c_p$ and the subscripts t and j represent time and depth, respectively. Combining like terms and rearranging so that all terms involving T_{j+1} are on the left-hand side of the equation, equation (2.13) becomes

$$\begin{aligned}T_{t+1,j-1}+\gamma_{t,j}T_{t+1,j}+T_{t+1,j+1}=\\ -T_{t,j-1}+T_{t,j}(\beta_{t,j}+\alpha_{t,j}-\delta_{t,j})+T_{t,j+1}(1-\beta_{t,j}+\delta_{t,j})+\mu_{t,j}\equiv\phi_{t,j}\end{aligned} \tag{2.14}$$

where

$$\begin{aligned}\alpha_{t,j}=-2\left(\frac{(\Delta z)^2}{k_{th_{t,j}}(\Delta t)}\right),\quad \gamma_{t,j}=\alpha_{t,j}-2,\quad \beta_{t,j}=2\left(\frac{k_{th_{t,j+1}}}{k_{th_{t,j}}}\right),\\ \delta_{t,j}=2v_{w_{t,j}}\left(\frac{c_{p,w}}{c_{p_{t,j}}}\frac{\Delta z}{k_{th_{t,j}}}\right),\quad \mu_{t,j}=\alpha_{t,j}\frac{l_{fus}}{c_p}\frac{\rho_i}{\rho_w}(\Delta\theta_i)_{t,j}.\end{aligned} \tag{2.15}$$

For more detail see Frankenstein and Koenig (2004a).

In the original version of FASST, the soil surface and foliage temperatures were solved for first. In order to solve equations (2.2) and (2.3), we (Frankenstein and Koenig 2004a) assumed that

$$
\begin{aligned}
T_g^4 &= \left(T_g\right)_{t-1}^4 + 4\left(T_g\right)_{t-1}^3\left[T_g - \left(T_g\right)_{t-1}\right] \\
q_{g,sat} &= q_{g,sat}\left(\left(T_g\right)_{t-1}\right) + \left.\frac{\partial q_{g,sat}}{\partial T_g}\right|_{T_g=\left(T_g\right)_{t-1}}\left[T_g - \left(T_g\right)_{t-1}\right]
\end{aligned}
\qquad (2.16)
$$

where the subscript "*t-1*" indicates the values of T_g at the previous time step. T_f^4 and $q_{f,sat}$ are also represented by equation (2.16), substituting the subscript g with f. The above substitutions allow the linearization of equations (2.2) and (2.3), which can then be solved simultaneously for T_f and T_g. The final equations are thus (Frankenstein and Koenig 2004b)

$$
\begin{aligned}
c_1^f + c_2^f T_g + c_3^f T_f &= 0; \\
c_1^g + c_2^g T_g + c_3^g T_f &= 0.
\end{aligned}
\qquad (2.17)
$$

2.6b Moisture

Equation (2.5) is solved numerically using an explicit scheme such that

$$
\frac{\theta_{w_{t+1,j}} - \theta_{w_{t,j}}}{\Delta t} = -\left[\frac{\left(v_w\right)_{t+1,j+1} - \left(v_w\right)_{t+1,j}}{\Delta z_j}\right] + sources(j) - losses(j) \qquad (2.18)
$$

where

$$
\begin{aligned}
\left(v_w\right)_j &= \left(K_{lh}\right)_{j-1/2}\left[\frac{h_j - h_{j-1}}{z_j - z_{j-1}}\right] \\
\left(v_w\right)_{j+1} &= \left(K_{lh}\right)_{j+1/2}\left[\frac{h_{j+1} - h_j}{z_{j+1} - z_j}\right].
\end{aligned}
\qquad (2.19)
$$

$h_j = z_j - \Psi_j$ and $\Delta z_j = (z_{j+1} - z_{j-1})/2$. The change in soil moisture content due to changes in the ice content, i.e., freezing/thawing, is incorporated into the source and sink terms. In Equation (2.5) it is the second term on the right-hand side. Reexpressing θ_w in terms of ψ using equation (2.7), equation (2.18) is solved for ψ using a Newton-Raphson technique.

3.0 New

3.1 Temperature/Energy

Equation (2.1) has been modified to include temperature changes due to vapor flow. It is now

$$\frac{\partial c_p T}{\partial t} - l_{fus}\rho_i \frac{\partial \theta_i}{\partial t} + l_w \rho_w \frac{\partial \theta_v}{\partial t} = \frac{\partial}{\partial z}\left(\kappa \frac{\partial T}{\partial z} - v_w c_{p,w} T - v_v c_{p,v}\left(T - 273.15\right) - l_w \rho_w v_v\right) + c_{p,w}\rho_w\left(sources - losses\right)T \tag{3.1}$$

where l_w is the latent heat of vaporization (J/kg), θ_v is the volumetric water vapor content (cm^3/cm^3), $c_{p,v}$ is the specific heat of water vapor ($J/kg{\cdot}K$), and v_v is the vertical rate of water vapor flow (m/s). Another change from the original formulation is that z (m) is now measured positive upwards from sea level.

The calculation of the specific heat, c_p, is slightly different than before due to the incorporation of water vapor into the soil matrix. The original equation was

$$c_p = (1-n)\gamma_d c_{p,s} + \theta_w \rho_w c_{p,w} + \left(n - \left[\theta_w + \theta_i\right]\right)\rho_a c_{p,a} + \theta_i \rho_i c_{p,i}\,. \tag{3.2}$$

It is now

$$c_p = (1-n)\gamma_d c_{p,s} + \theta_w \rho_w c_{p,w} + \left(n - \left[\theta_w + \theta_i + \theta_v\right]\right)\rho_a c_{p,a} + \theta_i \rho_i c_{p,i} + \theta_v \rho_v c_{p,v}\,. \tag{3.3}$$

See the list of variables at the beginning of the report for a description of all of the parameters. While the calculation of the thermal conductivity, κ, is the same as in the original version of FASST, the calculation of the individual components is different. Comparison of the original and new values of the thermal conductivity, specific heat, and density for water, air, ice, and water vapor are listed in Table 1. Unlike in the original version of FASST where we assumed that the specific heat and thermal conductivity of vegetation were negligible, I now assume that they are $c_{p,veg}$ = 3500 $J/kg{\cdot}K$ and κ_{veg} = 0.38 $W/m{\cdot}K$ (Moore and Fisch 1986).

Table 1. Physical properties of water, ice, air, and water vapor.

Variable	Old	New
$c_{p,w}$ T (°C) (J/kg·K)	4217.7	$-10^{-6}T^5 + 10^{-4}T^4 - 6\times10^{-3}T3 + 10^{-1}T^2 - 2.9T + 4216.9$ Hillel (1998) $4216.9 \le c_{p,w} \le 31500.0$
$c_{p,a}$ T (°C) (J/kg·K)	1005.6	$1.9327\times10^{-9}T^4 - 7.9999\times10^{-7}T^3 + 1.1407\times10^{-3}T^2$ $-0.4489T + 1057.5$ Ierardi (1999) $1000 \le c_{p,a} \le 1250$
$c_{p,i}$ (J/kg·K)	$-13.3 + 7.80T_a$	$-13.3 + 7.80T_a$ Jordan (1991) $1389 \le c_{p,i} \le 2050$
$c_{p,v}$ (J/kg·K)		$2.3888\times10^{-8}T^4 - 6.5129\times10^{-5}T^3 + 6.6178\times10^{-2}T^2$ $-2.9086\times10^{1}T + 6625.5$ Ierardi (1999) $2000 \le c_{p,v} \le 3260$
ρ_w (kg/m³)	1000.0	$-3.0\times10^{-7}T^4 + 7.0\times10^{-5}T^3 - 9.92\times10^{-3}T^2$ $+8.666\times10^{-2}T + 999.81$ Hillel (1998) $962 \le \rho_w \le 1000$
ρ_a (kg/m³)	$3.48\times10^{-3}\left(P_a/T_a\right)$	$360.77819T^{-1.00336}$ Ierardi (1999) $0.948 \le \rho_a \le 2.05$
ρ_i T (°C) (kg/m³)	916.5	$-2.0\times10^{-10}T^6 - 7.0\times10^{-8}T^5 - 1.0\times10^{-5}T^4$ $-7.0\times10^{-4}T^3 - 2.37\times10^{-2}T^2 - 4.36\times10^{-1}T^3 + 916.2$ Engineering ToolBox (2005) $916.2 \le \rho_i \le 925.7$
ρ_v (kg/m³)		$4.192\times10^{-12}T^4 - 1.25128\times10^{-8}T^3 + 1.45079\times10^{-5}T^2$ $-8.12253\times10^{-3}T + 2.17634$ Ierardi (1999) $0.60 \le \rho_v \le 1.14$
κ_w (W/m·K)	$1.8\times10^{-3}T - 0.0787$	$1.8\times10^{-3}T - 0.0787$ Farouki (1981) $0.58 \le \kappa_w \le 0.75$
κ_a (W/m·K)		$1.5207\times10^{-11}T^3 - 4.8574\times10^{-8}T^2 + 1.0184\times10^{-4}T$ -3.9333×10^{-4} Ierardi (1999) $0.0159 \le \kappa_a \le 0.1000$
κ_i (W/m·K)	$488.19T^{-1} + 0.4685$	$4.0\times10^{-7}T^3 + 1.0\times10^{-4}T^2 - 6.9\times10^{-3}T + 2.2174$ Engineering ToolBox (2005) $2.2174 \le \kappa_i \le 3.4800$
κ_v (W/m·K)		$8.3154\times10^{-5}T - 7.4556\times10^{-3}$ Ierardi (1999) $0.00695 \le \kappa_v \le 0.02360$

The new surface boundary conditions at the soil and vegetation surfaces are

$$F_f = 0 = \sigma_f \left[I_s^{\downarrow}(1-\alpha_f) + \varepsilon_f I_{ir}^{\downarrow} - \varepsilon_f \sigma T_f^4 - P_f \right] + \frac{\sigma_f \varepsilon_f \varepsilon_g \sigma}{\varepsilon_1}(T_g^4 - T_f^4)$$
$$+ H_f + L_f + \sigma_f \kappa_{veg} \frac{\partial T_f}{\partial z}. \tag{3.4}$$

$$F(T_g) = 0 = \left(1-\sigma_f\right)\left[I_s^{\downarrow}(1-\alpha_g) + \varepsilon_g I_{ir}^{\downarrow} - \varepsilon_g \sigma T_g^4 \right] - P_g - \frac{\sigma_f \varepsilon_f \varepsilon_g \sigma}{\varepsilon_1}(T_g^4 - T_f^4)$$
$$+ H_g + L_g + l_{fus} \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t} \Delta z - l_w \rho_w \frac{\partial \theta_v}{\partial t} + \kappa \frac{\partial T_g}{\partial z} - \left(v_w c_p + v_v c_{p,v}\right)\left(T_g - 273.15\right)$$
$$- l_w \rho_w v_v + c_{p,w} \rho_w \left(sources - losses\right) T_g. \tag{3.5}$$

Unlike in the original version of FASST where the bottom temperature was allowed to float, I assume a constant deep-earth heat flux for the bottom boundary condition of 75 mW/m^2.

3.2 Moisture/Mass

The water flow rate (v_w) is now

$$v_w = -K_{lh} \frac{\partial h}{\partial z} - K_{lT} \frac{\partial T}{\partial z} = -K_{lh}\left(1 + \frac{\partial \Psi}{\partial z}\right) - K_{lT} \frac{\partial T}{\partial z}. \tag{3.6}$$

where K_{lT} is the temperature-dependent hydraulic gradient ($m^2/K{\cdot}s$). The new moisture governing equation takes into account water flow due to temperature gradients, unlike the old version, which considered only flow due to gravity and pressure gradients. As with the temperature governing equations, vapor fluxes are also now accounted for. The new mass balance equations is

$$\frac{\partial \theta_w}{\partial t} + \frac{\partial \theta_v}{\partial t} + \frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial z}\left[K_{lh} - \frac{\partial \Psi}{\partial z}\left(K_{lh} + K_{vh}\right) + \frac{\partial T}{\partial z}\left(K_{lT} + K_{vT}\right) \right] + sources - losses \tag{3.7}$$

where K_{vT} is the temperature-dependent vapor gradient ($m^2/K{\cdot}s$), and K_{vh} is the pressure-dependent vapor conductivity (m/s). The surface moisture boundary condition remains unchanged from the original version of FASST.

Differences between equations (2.4) and (3.6) can be explained by the change in the vertical reference (z positive downward from the surface previously versus z positive upward from sea level currently) and the inclusion of temperature gradient flow.

3.3 Hydraulic Flow Parameters

The temperature-dependent hydraulic gradient, K_{lT}, is given as (Hansson et al. 2004, Noborio et al. 1996)

$$K_{lT} = K_{lh}\left(7\Psi \frac{1}{\gamma_o}\frac{d\gamma}{dT}\right). \tag{3.8}$$

$\gamma = 75.6 - 0.1425(T - 273.15) - 2.38x10^{-4}(T - 273.15)^2$ is the soil water surface tension (g/s^2), $\gamma_o = \gamma$ (25 °C) = 71.89 g/s^2, and K_{lh}, the pressure-driven hydraulic conductivity, is defined in equation (2.8).

The water vapor flow rate (v_v) is given by (Fayer, 2000)

$$v_v = -K_{vh}\frac{\partial\Psi}{\partial z} - K_{vT}\frac{\partial T}{\partial z} \tag{3.9}$$

where (Fayer 2000, Noborio et al. 1996, Nassar and Horton 1989)

$$K_{vh} = \frac{D\rho_{vs}Mg}{\rho_w RT}H_R \qquad D = 0.66\left[n - (\theta_w + \theta_i)\right]\left[2.12x10^{-5}\left(\frac{T}{273.15}\right)^2\right]$$
$$\eta = 9.5 + 6\theta_w - 8.5\exp\left\{-\left[\left(1 + 2.6m_c^{-1/2}\right)\theta_w\right]^4\right\} \tag{3.10}$$
$$K_{vT} = \frac{D\eta H_R}{\rho_w}\frac{d\rho_{vs}}{dT} \qquad H_R = \exp\left[-\frac{\Psi Mg}{RT}\right].$$

D is the molecular diffusivity of water vapor in air (m^2/s), n is the porosity, η is the mechanistic enhancement factor, m_c is the clay fraction, H_R is the soil relative humidity, M is the molecular weight of water (0.018015 kg/mol), R is the universal gas constant (8.314 $J/mol{\cdot}K$), and ρ_{vs} is the saturated water vapor density (kg/m^3).

3.4 Turbulent Energy Drag Coefficients

An effort was made to move away from a more empirical formulation of the turbulent energy terms and instead to base them on the more widely accepted Monin-Obukhov similarity theory. Unfortunately, this requires iterating for a solution, which is numerically cost-prohibitive. In order to avoid this, I adopted the method of Mascart et al. (1995) and Louis (1979). The bulk transfer coefficients C_{hn}^{g} and C_{en}^{g} are now defined as

$$C_{hn}^{g} = \frac{k^2/0.74}{\ln\left(Z_u/z_0^g\right)\ln\left(Z_u/z_0^h\right)}$$
$$C_{en}^{g} = \frac{k^2/0.74}{\ln\left(Z_u/z_0^g\right)\ln\left(Z_u/z_0^q\right)} \tag{3.11}$$

where z_o^h (m) is the sensible heat roughness length and z_o^q (m) is the moisture roughness length and all other terms are described in Section 2.3. The roughness lengths are calculated using (Jacobson 2005)

$$z_o^h = \begin{cases} z_o^g & u_* = 0.0 \\ \max\left(0.9 z_o^g, \dfrac{D_h}{k u_*}\right) & u_* \neq 0.0 \end{cases} \qquad D_h = \frac{\kappa_a}{\rho_a c_{p,w}}$$
$$if\ D_h \le 0.0, D_h = D_v$$
$$z_o^q = \begin{cases} z_o^g & u_* = 0.0 \\ \max\left(0.9 z_o^g, \dfrac{D_v}{k u_*}\right) & u_* \neq 0.0 \end{cases} \qquad D_v = 2.11x10^{-5}\left(\frac{T_a}{273.15}\right)^{1.94}\left(\frac{1013.25}{P_a}\right) \tag{3.12}$$

with (Louis 1979)

$$u_*^2 = C_{ng}^0 W^2 \Gamma_m \qquad C_{ng}^0 = \frac{k^2}{\left[\ln\left(Z_u / z_0^g\right)\right]^2}$$

$$\Gamma_m = \begin{cases} \left[1 - \dfrac{9.4 R_{ib}}{1 + C_m \left|R_{ib}\right|^{1/2}}\right] & R_{ib} \le 0 \\ \left[\dfrac{1}{1 + 4.7 R_{ib}}\right]^2 & R_{ib} > 0 \end{cases} \qquad C_m = 7.4 C_{ng}^0 9.4 \sqrt{\frac{Z_u}{z_0^g}} \tag{3.13}$$

where R_{ib}, the bulk Richardson number, is defined as

$$R_{ib} = \begin{cases} 0.21 & W_{af} < 0.01 \\ \dfrac{2 g Z_a \left(T_{af} - T_g\right)}{\left(T_{af} + T_g\right) W_{af}^2} & W_{af} \ge 0.01 \end{cases}$$

and T_{af} (K) and W_{af} (m/s) are the temperature and wind speed at the foliage/ground interface respectively (Deardorff 1978). The sensible heat exchange stability correction factor term (Γ_h) in equation (2.9) that accounts for non-neutral conditions is defined as (Mascart et al. 1995)

$$\Gamma_h = \begin{cases} \left[\dfrac{\ln\left(Z_u / z_0^g\right)}{\ln\left(Z_u / z_0^h\right)}\right]\left[1 - \dfrac{10 R_{ib}}{1 + C_h \left|R_{ib}\right|^{1/2}}\right] & R_{ib} \le 0 \\ \left[\dfrac{\ln\left(Z_u / z_0^g\right)}{\ln\left(Z_u / z_0^h\right)}\right]\left[\dfrac{1}{1 + 4.7 R_{ib}}\right]^2 & R_{ib} > 0 \end{cases} . \tag{3.14}$$

The latent heat exchange stability correction facter term, Γ_e, is the same as Γ_h, replacing z_o^h with z_o^q and C_h with C_e. C_h is defined as (Mascart et al. 1995)

$$
\begin{aligned}
C_h &= 9.4C_h^* C_{ng}^0 \left[\left[\frac{\ln\left(Z_u / z_0^g\right)}{\ln\left(Z_u / z_0^h\right)}\right]\right]\left(\frac{Z_u}{z_0^h}\right)^{p_h} \\
C_h^* &= 3.2165 + 4.3431\mu + 0.536\mu^2 - 0.0781\mu^3 \\
p_h &= 0.5802 - 0.1571\mu + 0.0327\mu^2 - 0.0026\mu^3 \\
\mu &= \ln\left(z_0^g / z_0^h\right).
\end{aligned}
\tag{3.15}
$$

C_e is similarly calculated by substituting z_o^h with z_o^q.

3.5 Precipitation Flux

In the new version of FASST, mixed precipitation is allowed. Thus, for a bare surface it is defined as

$$
P_g = -\left(\rho_w c_{p,w} \cdot \left[rain\ \ fallrate - I_r + D_r\right] + \gamma_p c_{p,i} \cdot \left[snow\ \ fallrate - I_s + D_s\right]\right) T_p
\tag{3.16}
$$

with T_p the precipitation temperature equal to the air wet-bulb temperature, I_r, I_s are respectively the vegetation intercepted rain and snow (m/s), and D_r, D_s are the vegetation dripped rain and snow (m/s), respectively.

3.6 Solution

Unlike the original version of FASST where the temperature profile was solved for before the moisture at a given time step, in the new version both profiles are iterated for simultaneously. Another change from the original version is that, since the orientation of the z-axis is reversed, node 1 is now at the bottom of the soil instead of the top. This allows the expansion of the solution matrix if snow is present.

3.6a Temperature

Following the technique outlined in Celia et al. (1990) and Hansson et al. (2004) the individual terms in the left-hand side of equation (3.1) are rewritten as

$$\frac{\partial c_p T}{\partial t} = \left[c_p\right]_{j,t+1/2} \left\{ \frac{\left[T\right]_{j,t+1}^{k+1} - \left[T\right]_{j,t}}{\Delta t} \right\} = \left[c_p\right]_{j,t+1/2} \left\{ \frac{\delta_j^T}{\Delta t} + \frac{\left[T\right]_{j,t+1}^{k} - \left[T\right]_{j,t}}{\Delta t} \right\} \tag{3.17}$$

$$\begin{aligned} -l_{fus}\rho_i \frac{\partial \theta_i}{\partial t} &= -\left[l_{fus}\rho_i\right]_{j,t+1/2} \left\{ \frac{\left[\theta_i\right]_{j,t+1}^{k+1} - \left[\theta_i\right]_{j,t}}{\Delta t} + \frac{\left[\theta_i\right]_{j,t+1}^{k+1} - \left[\theta_i\right]_{j,t}}{\Delta t} \right\} \\ &= \left[\frac{l_{fus}^2 \rho_i}{gT} \frac{d\theta_w}{d\Psi} \right]_{j,t+1}^{k} \left\{ \frac{\delta_j^T}{\Delta t} \right\} - \left[l_{fus}\rho_i\right]_{j,t+1/2} \left\{ \frac{\left[\theta_i\right]_{j,t+1}^{k+1} - \left[\theta_i\right]_{j,t}}{\Delta t} \right\} \end{aligned} \tag{3.18}$$

$$\begin{aligned} l_w \rho_w \frac{\partial \theta_v}{\partial t} &= \left[l_w \rho_w\right]_{j,t+1/2} \left\{ \frac{\left[\theta_v\right]_{j,t+1}^{k+1} - \left[\theta_v\right]_{j,t}}{\Delta t} + \frac{\left[\theta_v\right]_{j,t+1}^{k+1} - \left[\theta_v\right]_{j,t}}{\Delta t} \right\} \\ &= \left[l_w \rho_w\right]_{j,t+1/2} \left\{ \left[\frac{d\theta_v}{dT} \right]_{j,t+1}^{k} \left[\frac{\delta_j^T}{\Delta t} \right] + \frac{\left[\theta_v\right]_{j,t+1}^{k+1} - \left[\theta_v\right]_{j,t}}{\Delta t} \right\} \end{aligned} \tag{3.19}$$

where $\delta_j^T = T_{j,t+1}^{k+1} - T_{j,t+1}^{k}$ and the subscript "*j*" is depth and the superscript "*k*" is the iteration level. The right-hand side of equation (3.1) for the interior nodes becomes

$$\begin{aligned} &\frac{\partial}{\partial z}\left(\kappa \frac{\partial T}{\partial z} - v_w c_{p,w} T - v_v c_{p,v}\left(T - 273.15\right) - l_w \rho_w v_v \right) + c_{p,w}\rho_w\left(sources - losses\right)T \\ &= \frac{1}{\Delta z_j}\left\{ q_{j+1/2,t+1/2} - q_{j-1/2,t+1/2} \right\} + \left[\rho_w\right]_{j,t+1/2}\left(sources - losses\right)_{j,t+1/2}\left(\frac{\delta_j^T + T_{j,t+1}^k + T_{j,t}}{2} \right). \end{aligned} \tag{3.20}$$

Expanding only the $q_{j+1/2,t+1/2}$ term since the $q_{j-1/2,t+1/2}$ term is similar

$$q_{j+1/2,t+1/2} = \frac{1}{2} \left\{ \begin{aligned} &\kappa_{j+1/2,t+1}^k \left(\frac{\delta_{j+1}^T - \delta_j^T}{z_{j+1} - z_j} + \frac{T_{j+1,t+1}^k - T_{j,t+1}^k}{z_{j+1} - z_j} \right) + \kappa_{j+1/2,t}\left(\frac{T_{j+1,t} - T_{j,t}}{z_{j+1} - z_j} \right) \\ &-\left[v_w c_{p,w} + v_v c_{p,v} \right]_{j+1/2,t+1}^k \left(\frac{\delta_{j+1}^T + \delta_j^T}{2} + \frac{T_{j+1,t+1}^k + T_{j,t+1}^k}{2} - 273.15 \right) \\ &-\left[v_w c_{p,w} + v_v c_{p,v} \right]_{j+1/2,t} \left(\frac{T_{j+1,t} + T_{j,t}}{2} - 273.15 \right) \\ &-\left[l_w \rho_w v_v \right]_{j+1/2,t+1}^k - \left[l_w \rho_w v_v \right]_{j+1/2,t} \end{aligned} \right\}. \tag{3.21}$$

For the node at the bottom of the soil column (j = 1), $q_{j-1/2,t+1/2}$ is

$$q_{j-1/2,t+1/2} = \frac{1}{2}\left\{\begin{array}{l} 2(0.075) - \left[v_w c_{p,w} + v_v c_{p,v}\right]_{j,t+1}^k \left(\frac{\delta_j^T}{2} + \frac{T_{j,t+1}^k}{2} - 273.15\right) \\ -\left[v_w c_{p,w} + v_v c_{p,v}\right]_{j,t} \left(\frac{T_{j,t}}{2} - 273.15\right) - \left[l_w \rho_w v_v\right]_{j,t+1}^k - \left[l_w \rho_w v_v\right]_{j,t} \end{array}\right\}, \quad (3.22)$$

while for the surface node (j = nnodes) without vegetation it is

$$q_{j+1/2,t+1/2} = \frac{1}{2}\left\{\begin{array}{l} \left[I_s^{\downarrow}(1-\alpha_g) + \varepsilon_g I_{ir}^{\downarrow} - \varepsilon_g \sigma T_j^4 - P_g + H_g + L_g\right]_{j,t+1}^k \\ +\left[I_s^{\downarrow}(1-\alpha_g) + \varepsilon_g I_{ir}^{\downarrow} - \varepsilon_g \sigma T_j^4 - P_g + H_g + L_g\right]_{j,t} \\ -\left[v_w c_{p,w} + v_v c_{p,v}\right]_{j,t+1}^k \left(\frac{\delta_j^T}{2} + \frac{T_{j,t+1}^k}{2} - 273.15\right) \\ -\left[v_w c_{p,w} + v_v c_{p,v}\right]_{j,t} \left(\frac{T_{j,t}}{2} - 273.15\right) - \left[l_w \rho_w v_v\right]_{j,t+1}^k - \left[l_w \rho_w v_v\right]_{j,t} \end{array}\right\}. \quad (3.23)$$

If vegetation is present, then equation (3.20) for the vegetation (j = nnodes + 1) and surface (j = nnodes) nodes is written respectively as

$$q_{j+1/2,t+1/2} - q_{j-1/2,t+1/2} = \frac{1}{2}\left\{\begin{array}{l} \left[\sigma_f\left[I_s^{\downarrow}(1-\alpha_f)+\varepsilon_f I_{ir}^{\downarrow}-\varepsilon_f\sigma T_f^4-P_f\right]+\dfrac{\sigma_f\varepsilon_f\varepsilon_g\sigma}{\varepsilon_1}(T_{j-1}^4-T_j^4)+H_f+L_f\right]_{j,t+1}^k \\ +\left[\sigma_f\left[I_s^{\downarrow}(1-\alpha_f)+\varepsilon_f I_{ir}^{\downarrow}-\varepsilon_f\sigma T_f^4-P_f\right]+\dfrac{\sigma_f\varepsilon_f\varepsilon_g\sigma}{\varepsilon_1}(T_{j-1}^4-T_j^4)+H_f+L_f\right]_{j,t} \\ -\left[\sigma_f\kappa\right]_{j-1/2,t+1}^k\left(\dfrac{\delta_j^T-\delta_{j-1}^T}{z_j-z_{j-1}}+\dfrac{T_{j,t+1}^k-T_{j-1,t+1}^k}{z_j-z_{j-1}}\right)-\left[\sigma_f\kappa\right]_{j-1/2,t}\left(\dfrac{T_{j,t}-T_{j-1,t}}{z_j-z_{j-1}}\right) \end{array}\right\}$$

$$q_{j+1/2,t+1/2} - q_{j-1/2,t+1/2} = \frac{1}{2}\left\{\begin{array}{l} \left[\left(1-\sigma_f\right)\left[I_s^{\downarrow}(1-\alpha_g)+\varepsilon_g I_{ir}^{\downarrow}-\varepsilon_g\sigma T_g^4\right]-P_g-\dfrac{\sigma_f\varepsilon_f\varepsilon_g\sigma}{\varepsilon_1}(T_j^4-T_{j+1}^4)+H_g+L_g\right]_{j,t+1}^k \\ +\left[\left(1-\sigma_f\right)\left[I_s^{\downarrow}(1-\alpha_g)+\varepsilon_g I_{ir}^{\downarrow}-\varepsilon_g\sigma T_g^4\right]-P_g-\dfrac{\sigma_f\varepsilon_f\varepsilon_g\sigma}{\varepsilon_1}(T_j^4-T_{j+1}^4)+H_g+L_g\right]_{j,t} \\ +\left[\sigma_f\kappa\right]_{j+1/2,t+1}^k\left(\dfrac{\delta_{j+1}^T-\delta_j^T}{z_{j+1}-z_j}+\dfrac{T_{j+1,t+1}^k-T_{j,t+1}^k}{z_{j+1}-z_j}\right)+\left[\sigma_f\kappa\right]_{j+1/2,t}\left(\dfrac{T_{j+1,t}-T_{j,t}}{z_{j+1}-z_j}\right) \\ -\left[v_w c_{p,w}+v_v c_{p,v}\right]_{j,t+1}^k\left(\dfrac{\delta_j^T}{2}+\dfrac{T_{j,t+1}^k}{2}-273.15\right)-\left[v_w c_{p,w}+v_v c_{p,v}\right]_{j,t}\left(\dfrac{T_{j,t}}{2}-273.15\right) \\ -\left[l_w\rho_w v_v\right]_{j,t+1}^k-\left[l_w\rho_w v_v\right]_{j,t} \end{array}\right. \qquad (3.24)$$

Substituting equations (3.19)–(3.24) into equation (3.1), assuming (2.16) is still valid and rearranging so that all δ^T terms are on the left-hand side, the final temperature equation is

$$A_j^T\delta_{j-1}^T + B_j^T\delta_j^T + C_j^T\delta_{j+1}^T = D_j^T\,. \qquad (3.25)$$

A Newton-Raphson technique is used to solve the system of linear equations as with the original version of FASST.

3.6b Moisture

Similar to the solution technique for the temperature equation, I follow the technique outlined in Celia et al. (1990). Equation (3.7) is thus

$$\begin{aligned}\frac{\partial\theta}{\partial t} &= -\frac{\partial}{\partial z}\{v\} + sources - sinks \\ &= -\frac{1}{\Delta z_j}\left\{v_{j+1/2,t+1/2} - v_{j-1/2,t+1/2}\right\} + \left(sources - sinks\right)_{j,t+1/2}\end{aligned} \tag{3.26}$$

where $\frac{\partial\theta}{\partial t} = \frac{\partial\theta_w}{\partial t} + \frac{\partial\theta_v}{\partial t} + \frac{\rho_i}{\rho_w}\frac{\partial\theta_i}{\partial t}$, $v = v_w + v_v$ and $v_{j\pm1/2,t+1/2} = \frac{1}{2}\left(v_{j\pm1/2,t+1} + v_{j\pm1/2,t}\right)$.

At the surface, $v_{j+1/2,t} = q_{top}$ while at the bottom, $v_{j-1/2,t} = q_{bot}$, both of which are given in equation (2.6). Expanding the left-hand side of equation (3.26)

$$\begin{aligned}\frac{\partial[\theta_w]_j}{\partial t} &= \frac{[\theta_w]^{k+1}_{j,t+1} - [\theta_w]_{j,t}}{\Delta t} = \frac{[\theta_w]^{k+1}_{j,t+1} - [\theta_w]^{k}_{j,t+1} + [\theta_w]^{k}_{j,t+1} - [\theta_w]_{j,t}}{\Delta t} \\ &= \left.\frac{d\theta_w}{d\Psi}\right|^{k}_{j,t+1} \frac{\Psi^{k+1}_{j,t+1} - \Psi^{k}_{j,t+1}}{\Delta t} + \frac{[\theta_w]^{k}_{j,t+1} - [\theta_w]_{j,t}}{\Delta t} \\ &= \left.\frac{d\theta_w}{d\Psi}\right|^{k}_{j,t+1} \frac{\delta^{\Psi}_j}{\Delta t} + \frac{[\theta_w]^{k}_{j,t+1} - [\theta_w]_{j,t}}{\Delta t} \\ \frac{\partial\theta_v}{\partial t} + \frac{\rho_i}{\rho_w}\frac{\partial\theta_i}{\partial t} &= \frac{[\theta_v]^{k}_{j,t+1} - [\theta_v]_{j,t}}{\Delta t} + \left[\frac{\rho_i}{\rho_w}\right]_{j,t+1/2} \frac{[\theta_i]^{k}_{j,t+1} - [\theta_i]_{j,t}}{\Delta t}\end{aligned} \tag{3.27}$$

where “k” is the iteration level and $\delta^{\Psi}_j = \Psi^{k+1}_{j,t+1} - \Psi^{k}_{j,t+1}$. The $[v_w]_{j+1/2,t+1/2}$ term in the right-hand side of equation (3.26) becomes

$$[v_w]_{j+1/2,t+1/2} = -\frac{1}{2}\left\{\begin{aligned}&[K_{lh}]^{k}_{j+1/2,t+1}\left(\frac{\delta^{\Psi}_{j+1} - \delta^{\Psi}_{j}}{z_{j+1} - z_j} + \frac{\Psi^{k}_{j+1,t+1} - \Psi^{k}_{j,t+1}}{z_{j+1} - z_j} + 1\right) \\ &+ [K_{lT}]^{k}_{j+1/2,t+1}\left(\frac{T^{k}_{j+1,t+1} - T^{k}_{j,t+1}}{z_{j+1} - z_j}\right) \\ &+ [K_{lh}]_{j+1/2,t}\left(\frac{\Psi_{j+1,t} - \Psi_{j,t}}{z_{j+1} - z_j} + 1\right) + [K_{lT}]_{j+1/2,t}\left(\frac{T_{j+1,t} - T_{j,t}}{z_{j+1} - z_j}\right)\end{aligned}\right\}. \tag{3.28}$$

Except for the “+ 1” terms in equation (3.28), the expression for $[v_v]_{j+1/2,t+1/2}$ is the same. The same principle is applied to the $[v]_{j-1/2,t+1/2}$ term in equation (3.26). Substituting equations (3.27) and (3.28) into equation (3.26) and rearranging so that all δ^{Ψ} terms are on the left-hand side, the final moisture equation is

$$A_j^{\Psi}\delta_{j-1}^{\Psi} + B_j^{\Psi}\delta_j^{\Psi} + C_j^{\Psi}\delta_{j+1}^{\Psi} = D_j^{\Psi} \; . \qquad 3.29$$

As with the original version of FASST, a Newton-Raphson technique is used to solve the system of linear equations.

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14. ABSTRACT

This paper discusses only the differences between the original version of FASST (Frankenstein and Koenig 2004a, 2004b) and the new version. This report is intended as a supplement to the original model documentation. In its original incarnation, energy and mass transport associated with water vapor in the soil matrix were ignored. The author added these so that model usage could be expanded to include biological investigations yet still retain its initial focus of soil strength, and sensor performance inputs. Also ignored in the original version was water transport due to soil temperature gradients. In determining the new soil temperatures and moistures, the original model first achieved convergence in the temperature profile followed by the moisture profile at a given time step. The new version of FASST solves both of these sets of equations simultaneously. No changes have been made to the equations describing the canopy physical state except to allow for mixed precipitation.

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